Baryon Magnetic Moments in a Relativistic Quark Model

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Abstract

Magnetic moments of baryons in the ground-state octet and decuplet are calculated in a light-front framework. We investigate the effects of quark mass variation both in the current operator and in the wavefunctions. A simple fit uses single oscillator wavefunctions for the baryons and allows the three flavors of quark to have nonzero anomalous magnetic moments. We find a good fit to the data without allowing for strange quark contributions to the nucleon moments. A slightly better fit is obtained by allowing for explicit SU(3)$_f$ breaking in the wavefunctions through a simple mechanism. The predictions for magnetic moments in our relativistic model are also much less sensitive to the values chosen for the constituent quark masses than those of nonrelativistic models. Relativistic effects can be of order 20% in general, and can alter familiar relationships between the moments based on SU(3)$_f$ and a nonrelativistic treatment of spin.

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I. INTRODUCTION

In the simple additive quark model without sea quark degrees of freedom, and in the nonrelativistic limit, the magnetic moments of the baryons considered here are given by

$$\begin{aligned}
&\mu_p = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d; \quad \mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u \\
&\mu_\Lambda = \mu_s \\
&\mu_{\Sigma^+} = \frac{4}{3}\mu_u - \frac{1}{3}\mu_s; \quad \mu_{\Sigma^0\to\Lambda^0} = -\frac{1}{\sqrt{3}}(\mu_u - \mu_d); \quad \mu_{\Sigma^-} = \frac{4}{3}\mu_d - \frac{1}{3}\mu_s \\
&\mu_{\Xi^0} = \frac{4}{3}\mu_s - \frac{1}{3}\mu_u; \quad \mu_{\Xi^-} = \frac{4}{3}\mu_s - \frac{1}{3}\mu_d \qquad (1) \\
&\mu_{\Delta^{++}} = 3\mu_u; \quad \mu_{\Delta^+} = 2\mu_u + \mu_d; \quad \mu_{\Delta^0} = 2\mu_d + \mu_u; \quad \mu_{\Delta^-} = 3\mu_d \\
&\mu_{\Sigma^{*+}} = 2\mu_u + \mu_s; \quad \mu_{\Sigma^{*0}} = \mu_u + \mu_d + \mu_s; \quad \mu_{\Sigma^{*-}} = 2\mu_d + \mu_s \\
&\mu_{\Xi^{*0}} = 2\mu_s + \mu_u; \quad \mu_{\Xi^{*-}} = 2\mu_s + \mu_d \\
&\mu_{\Omega^-} = 3\mu_s.
\end{aligned}$$

These equations are generalized by Karl [1] to include possible contributions of a polarized strange quark sea to the magnetic moments of the nucleons. Since his model is based on the assumption of $\mathrm{SU}(3)_f$ symmetry among the ground state baryons, the presence of this polarized strange quark sea implies additional contributions to the moments of other baryons. These generalized Sehgal equations [2] reduce to Eqs. (1) by using the nonrelativistic quark model values of $\Delta u = \frac{4}{3}$, $\Delta d = -\frac{1}{3}$, and $\Delta s = 0$, where the Δq are the contributions of the quarks (and of antiquarks of the same flavor, in general) of a given flavor to the axial-vector current of the proton for which $g_A = \Delta u - \Delta d = 5/3$. The $\Sigma^0 \to \Lambda^0$ moment is a transition magnetic moment which is extracted from measurements of the amplitudes for the decay $\Sigma^0 \to \Lambda^0\gamma$ (see Ref. [3] and Appendix A). Note that the physical Σ^0 and Λ^0 states are not pure flavor eigenstates but are mixed by isospin-breaking interactions, and we consider the effects of this mixing on both the Λ^0 moment and this transition moment. Of the decuplet moments only those of the Δ^{++} and Ω^- are known; the Δ^{++} moment is extracted (with some uncertainty) from π^+p bremsstrahlung data [4].

A constituent quark model represents a significant truncation of a Lagrangian field theory like QCD, but it captures many important degrees of freedom and permits a systematic analysis of a large body of data. The parameters of the model should be interpreted as those of the original current quarks, but substantially dressed by nonperturbative effects of QCD. Thus, one would expect that the quark masses and other properties such as their magnetic moments would be substantially renormalized.

Eqs. (1) allow the inclusion of anomalous magnetic moments for the quarks, if the three quark moments μ_u, μ_d, and μ_s are considered as free parameters. Note that the effective additive magnetic moment of a strange quark is assumed to be the same, for example, in the Λ^0 state (where the other quarks are light) and in the Ω^- state (composed entirely of strange quarks). As pointed out by Karl, this assumption may not hold if the sizes of the baryons are dependent on their quark structure [which introduces explicit breaking of $\mathrm{SU}(3)_f$]. It is also possible that relativistic effects, both kinematical and dynamical, can modify these relations. Our calculation allows us to explore these possibilities in a simple way.

Previous relativistic work based on light-front dynamics [5–9] has shown that it is impossible to fit simultaneously the proton and neutron magnetic moments without some sort of modification of the quark model parameters. These calculations were also carried out in the absence of a strange quark contribution to the proton electromagnetic currents. They employed a Gaussian wave function for the nucleons of the form $\exp[-M_0^2/\beta^2]$, where M_0 is the non-interacting mass of the three-quark system and β is a size parameter [10], which permits simple analytic calculations but which cannot easily be extended to a complete set of orthonormal wave functions which can be mixed via a realistic interaction. Aznauryan, et al., considered magnetic moments and weak decay constants in the baryon octet, varying the anomalous quark moments to achieve a fit [5]. Tupper, et al. [6], Chung and Coester [8], and Cardarelli, et al. [9] examined the sensitivity of the fits to variations in the quark parameters (mass, anomalous moment, etc.). For some reasonable values of the baryon parameters, this meant adopting anomalous moments for the light quarks which are not proportional to their charges. Schlumpf calculated magnetic moments and weak decay constants in the

baryon octet [7] and decuplet [11], and varied the size parameter β separately for the two subclusters in the three-quark system, i.e., a quark-diquark picture.

Our approach is similar to several of the earlier works cited above, except that we provide further generality to the calculations. In particular, we study the effects of unequal quark masses, not just in the quark magnetic moments, but in the wave functions and current matrix elements. The spatial wavefunctions are oscillator ground states characterized at first by a single momentum parameter β, but then are given two momentum parameters which depend kinematically upon the quark masses.

We calculate magnetic moments in a light-front framework for the entire ground state baryon octet and decuplet. We also discuss our results in light of some recent direct calculations of quark anomalous magnetic moments via meson loops.

II. OUTLINE OF CALCULATION

The elements of the calculation of baryon light-front current matrix elements are described in detail in Ref. [12]. We present here a brief summary of the important features of the calculation of matrix elements and the extraction of magnetic moments.

Free-particle state vectors $|\tilde{\mathbf{p}}\mu\rangle$ are labeled by the light-front vector $\tilde{\mathbf{p}} = (p^+, \mathbf{p}_\perp)$ and are normalized as follows:

$$\langle \tilde{\mathbf{p}}'\mu' | \tilde{\mathbf{p}}\mu \rangle = (2\pi)^3 \delta_{\mu'\mu} \delta(\tilde{\mathbf{p}}' - \tilde{\mathbf{p}}). \tag{2}$$

A calculation of the matrix elements

$$\langle Mj; \tilde{\mathbf{P}}'\mu' | I^+(0) | Mj; \tilde{\mathbf{P}}\mu \rangle \rightarrow \langle \tilde{\mathbf{P}}'\mu' | I^+(0) | \tilde{\mathbf{P}}\mu \rangle \tag{3}$$

is sufficient to determine all Lorentz-invariant form factors for a baryon of mass M and spin j. The current operator is taken to be the sum of single-quark operators with light-front spinor matrix elements given by

$$\langle \tilde{\mathbf{p}}'\mu' | I^+(0) | \tilde{\mathbf{p}}\mu \rangle = F_{1q}(Q^2)\delta_{\mu'\mu} - i(\sigma_y)_{\mu'\mu} \frac{Q}{2m} F_{2q}(Q^2), \tag{4}$$

where $Q \simeq \sqrt{-q^\nu q_\nu}$. The momentum wavefunctions are expressed as follows:

$$
\begin{aligned}
\langle \tilde{\mathbf{p}}_1\mu_1\tilde{\mathbf{p}}_2\mu_2\tilde{\mathbf{p}}_3\mu_3|\tilde{\mathbf{P}}\mu\rangle =& \left|\frac{\partial(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{p}}_3)}{\partial(\tilde{\mathbf{P}},\mathbf{k}_1,\mathbf{k}_2)}\right|^{-\frac{1}{2}}(2\pi)^3\delta(\tilde{\mathbf{p}}_1+\tilde{\mathbf{p}}_2+\tilde{\mathbf{p}}_3-\tilde{\mathbf{P}})\\
&\times\langle \tfrac{1}{2}\bar{\mu}_1\tfrac{1}{2}\bar{\mu}_2|s_{12}\mu_{12}\rangle\langle s_{12}\mu_{12}\tfrac{1}{2}\bar{\mu}_3|s\mu_s\rangle\langle l_\rho\mu_\rho l_\lambda\mu_\lambda|L\mu_L\rangle\langle L\mu_L s\mu_s|j\mu\rangle\\
&\times Y_{l_\rho\mu_\rho}(\hat{\mathbf{k}}_\rho)Y_{l_\lambda\mu_\lambda}(\hat{\mathbf{K}}_\lambda)\Phi(k_\rho,K_\lambda)\\
&\times D^{(\frac{1}{2})\dagger}_{\bar{\mu}_1\mu_1}[\underline{R}_{cf}(k_1)]D^{(\frac{1}{2})\dagger}_{\bar{\mu}_2\mu_2}[\underline{R}_{cf}(k_2)]D^{(\frac{1}{2})\dagger}_{\bar{\mu}_3\mu_3}[\underline{R}_{cf}(k_3)],
\end{aligned}
\tag{5}
$$

where the μ_i are light-front quark spin projections, $\tilde{\mathbf{p}}_i$ the light-front quark momenta, $\Phi(k_\rho, K_\lambda)$ is the orbital momentum wavefunction, and $\underline{R}_{cf}(k_3)$ is a Melosh rotation:

$$
\underline{R}_{cf}(k) = \frac{(p^+ + m) - i\,\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\times\mathbf{p}_\perp}{[2p^+(p^0+m)]^{\frac{1}{2}}}. \tag{6}
$$

The quantum numbers of the state vectors correspond to irreducible representations of the permutation group. The spins (s_{12}, s) can have the values $(0, \frac{1}{2})$, $(1, \frac{1}{2})$ and $(1, \frac{3}{2})$, corresponding to quark-spin wavefunctions with mixed symmetry (χ^ρ and χ^λ) and total symmetry (χ^S), respectively. The momenta

$$
\begin{aligned}
\mathbf{k}_\rho \equiv& \frac{1}{\sqrt{2}}(\mathbf{k}_1 - \mathbf{k}_2)\\
\mathbf{K}_\lambda \equiv& \frac{1}{\sqrt{6}}(\mathbf{k}_1 + \mathbf{k}_2 - 2\mathbf{k}_3)
\end{aligned}
\tag{7}
$$

preserve the appropriate symmetries under various exchanges of $\mathbf{k}_1$, $\mathbf{k}_2$ and $\mathbf{k}_3$, the quark three-momenta in the baryon rest frame.

The set of state vectors formed using Eq. (5) and Gaussian functions of the momentum variables defined in Eq. (7) is complete and orthonormal. Since they are eigenfunctions of the overall spin, they satisfy the relevant rotational covariance properties. For this work, the spatial wavefunctions are taken to be oscillator ground states of the form

$$
\Phi(k_\rho, K_\lambda) = \frac{1}{\pi^{\frac{3}{2}}\beta_\rho^{\frac{3}{2}}\beta_\lambda^{\frac{3}{2}}} e^{-\left(\frac{k_\rho^2}{2\beta_\rho^2}+\frac{K_\lambda^2}{2\beta_\lambda^2}\right)}, \tag{8}
$$

where β_ρ and β_λ are for the moment set equal to a single parameter for all of the states considered. Later we consider the effects of allowing β_ρ and β_λ to vary by means of a simple

(nonrelativistic harmonic oscillator) formula in terms of the masses of the quarks involved. We will show in what follows that the dependence of our results for magnetic moments on the form of the spatial wavefunctions (through the oscillator parameters) is weak, which partially justifies our use of these simple wavefunctions.

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While the matrix elements of $I^+(0)$ are sufficient to determine all form factors, they are in fact not independent of each other. Parity considerations imply that

$$\langle \tilde{\mathbf{P}}' - \mu' | I^+(0) | \tilde{\mathbf{P}} - \mu \rangle = (-1)^{\mu' - \mu} \langle \tilde{\mathbf{P}}' \mu' | I^+(0) | \tilde{\mathbf{P}} \mu \rangle \tag{9}$$

This cuts the number of independent matrix elements in half. For elastic scattering, time-reversal symmetry provides another constraint:

$$\langle \tilde{\mathbf{P}}' \mu | I^+(0) | \tilde{\mathbf{P}} \mu' \rangle = (-1)^{\mu' - \mu} \langle \tilde{\mathbf{P}}' \mu' | I^+(0) | \tilde{\mathbf{P}} \mu \rangle \tag{10}$$

In addition, there can be constraints which come from the requirement of rotational covariance of the current operator. These can be expressed in terms of relations among the matrix elements of $I^+(0)$ [13]:

$$\sum_{\lambda' \lambda} D^{j\dagger}_{\mu' \lambda'}(\underline{R}'_{ch}) \langle \tilde{\mathbf{P}}' \lambda' | I^+(0) | \tilde{\mathbf{P}} \lambda \rangle D^j_{\lambda \mu}(\underline{R}_{ch}) = 0, \quad |\mu' - \mu| \geq 2, \tag{11}$$

where

$$\underline{R}_{ch} = \underline{R}_{cf}(\tilde{\mathbf{P}}, M) \underline{R}_y(\frac{\pi}{2}), \quad \underline{R}'_{ch} = \underline{R}_{cf}(\tilde{\mathbf{P}}', M) \underline{R}_y(\frac{\pi}{2}). \tag{12}$$

For $J^\pi = \frac{1}{2}^+$ baryons, there are four matrix elements of $I^+(0)$, of which only two are independent due to parity symmetry. Because $j = \frac{1}{2}$, there is no nontrivial constraint due to rotational covariance. The baryon form factors F_1 and F_2 are determined directly via

$$\langle \tilde{\mathbf{P}}' \mu' | I^+(0) | \tilde{\mathbf{P}} \mu \rangle = F_1(Q^2) \delta_{\mu' \mu} - i(\sigma_y)_{\mu' \mu} \frac{Q}{2M} F_2(Q^2). \tag{13}$$

For $J^\pi = \frac{3}{2}^+$ baryons, there are 16 matrix elements of $I^+(0)$, of which eight are independent due to parity symmetry. Time-reversal symmetry eliminates two more matrix elements. Thus, there are six independent matrix elements of I^+, which can be chosen without loss of generality to be $\langle +\frac{3}{2}|I^+(0)|\pm\frac{3}{2}\rangle$, $\langle +\frac{3}{2}|I^+(0)|\pm\frac{1}{2}\rangle$ and $\langle +\frac{1}{2}|I^+(0)|\pm\frac{1}{2}\rangle$. There are three constraints due to rotational covariance, but one of these is redundant due to time-reversal symmetry. Thus, only four of the above six matrix elements should be truly independent under rotational symmetry. For the results reported here, we choose the matrix elements $\langle +\frac{1}{2}|I^+(0)|\pm\frac{1}{2}\rangle$, $\langle +\frac{3}{2}|I^+(0)|+\frac{1}{2}\rangle$ and $\langle +\frac{3}{2}|I^+(0)|+\frac{3}{2}\rangle$, since they correspond to the lowest light-front spin transfer values.

Full rotational symmetry is a dynamical constraint on light-front calculations, and can only be fully satisfied by introducing two-body current matrix elements. However, the constraint due to rotational covariance is proportional to Q^2 as $Q^2 \to 0$. This means that it has no effect on calculations of magnetic moments. Nevertheless, there can still be relativistic effects of $O(Q)$, *i.e.*, to arbitrary order in $1/m$.

Extraction of the magnetic moment of $J^P = \frac{3}{2}^+$ states proceeds by calculation of the relativistic canonical-spin matrix elements of the operator $i\mathbf{S}_X \times \mathbf{q}$, where $\mathbf{S}_X$ is the spin of the baryon $X \in \{\Delta, \Omega\}$. The Sachs form factors of the X state are defined in terms of canonical spins by the relation [14]

$$\begin{aligned} {}_c\langle Xp's'|\mathbf{I}(0)|Xps\rangle_c = \frac{e}{2M_X}\psi^\dagger_{X,s'}\Bigg[\left(G^X_{E0} + G^X_{E2}\left[\mathbf{S}^{[2]} \times \mathbf{q}^{[1]} \times \mathbf{q}^{[1]}\right]^{[0]}\right)\mathbf{P} \\ + i\left(G^X_{M1}\mathbf{S}_X + G^X_{M3}\left[\mathbf{S}^{[3]}_X \times \left[\mathbf{q}^{[1]} \times \mathbf{q}^{[1]}\right]^{[2]}\right]^{[1]}\right) \times \mathbf{q}\Bigg]\psi_{X,s}, \end{aligned} \tag{14}$$

and the reduced matrix elements of the spin operators

$$\begin{aligned} {}_c\langle \tfrac{3}{2}||\mathbf{S}_X||\tfrac{3}{2}\rangle_c &= 2\sqrt{15}, \\ {}_c\langle \tfrac{3}{2}||\mathbf{S}^{[2]}_X||\tfrac{3}{2}\rangle_c &= -\sqrt{10/3}, \\ {}_c\langle \tfrac{3}{2}||\mathbf{S}^{[3]}_X||\tfrac{3}{2}\rangle_c &= -(7/3)\sqrt{2/3} \end{aligned} \tag{15}$$

and $q^{[2]} = [q^{[1]} \times q^{[1]}]^{[2]} = \sqrt{\frac{8\pi}{15}}\mathbf{q}^2 Y_{[2]}(\Omega_q)$. For $\lambda = -\frac{1}{2}$, $\lambda' = +\frac{1}{2}$, and momentum transfer along the z axis (conventional Breit frame),

$$_c\langle \tfrac{3}{2} + \tfrac{1}{2}| i[\mathbf{S}_X \times \mathbf{q}]_{1\mu} | \tfrac{3}{2} - \tfrac{1}{2}\rangle_c = -4\sqrt{2}Q. \tag{16}$$

Thus, for the +1 spherical tensor component of the current [15],

$$-\frac{1}{\sqrt{2}}{}_c\langle \tfrac{3}{2} + \tfrac{1}{2}| I^1(0) + iI^2(0) | \tfrac{3}{2} - \tfrac{1}{2}\rangle_c = -4\sqrt{2}\,\frac{Q}{2M_X}\,G_{M1}. \tag{17}$$

III. RESULTS

As a direct measure of the size of relativistic effects in the calculation of the baryon magnetic moments, we show in Table I a comparison between magnetic moments calculated using the nonrelativistic formulae of Eqs. (1) and our relativistic approach. We have restricted this calculation to baryons for which moment data exist, using the quark masses $m_{u,d} = 330$ MeV and $m_s = 550$ MeV [chosen roughly to fit the moments using the nonrelativistic formulae in Eqs. (1)] and $\beta_\rho = \beta_\lambda = 0.41$ GeV in our relativistic calculation, and have found the corresponding nonrelativistic moments using Eqs. (1). This value of harmonic oscillator parameter has been shown roughly to fit the nucleon form factors when calculated in a relativistic model with single-oscillator wavefunctions in Ref. [12]. Note that the relativistic calculation uses the physical mass for the baryon, rather than the sum of the quark masses, when calculating kinematical quantities.

Interactions between the quarks which distinguish between the u and d quarks can cause an isospin-violating mixing between the Σ^0 and Λ^0. As the two-state mixing angle $\theta_{\Sigma\Lambda}$ is about +0.0135 radians [17], the physical states Σ^0 and Λ^0 can to a good approximation be written in terms of the SU(3)$_f$ flavor eigenstates $\bar{\Sigma}^0$ and $\bar{\Lambda}^0$ as

$$\begin{aligned} \Sigma^0 &= \bar{\Sigma}^0 - \theta_{\Sigma\Lambda}\bar{\Lambda}^0 \\ \Lambda^0 &= \bar{\Lambda}^0 + \theta_{\Sigma\Lambda}\bar{\Sigma}^0. \end{aligned} \tag{18}$$

This mixing affects the Λ^0 moment as well as the $\Sigma^0 \to \Lambda^0$ transition moment (and in principle also the unmeasured Σ^0 moment) by

$$\mu_{\Lambda^0} = \mu_{\bar{\Lambda}^0} + 2\theta_{\Sigma\Lambda}\mu_{\bar{\Sigma}^0\to\bar{\Lambda}^0}$$

$$\mu_{\Sigma^0\to\Lambda^0} = \mu_{\bar{\Sigma}^0\to\bar{\Lambda}^0} + \theta_{\Sigma\Lambda}\left(\mu_{\bar{\Sigma}^0} - \mu_{\bar{\Lambda}^0}\right). \tag{19}$$

The results shown for μ_{Λ^0} and $\mu_{\Sigma^0\to\Lambda^0}$ in Table I (and below in Table II) are inclusive of the mixing correction to the flavor eigenstate moments which we calculate in our model. The effect is to lower μ_{Λ^0} by about -0.04 μ_N, and raise $\mu_{\Sigma^0\to\Lambda^0}$ by between 0.01 and 0.02 μ_N.

The second column in Table II shows the result of reducing the constituent quark masses to $m_{u,d} = 220$ MeV and $m_s = 419$ MeV, which are the values which fit the meson and baryon spectra [18,19] and, more importantly, the mass splittings between the various charge states [20] in the relativized quark model. One clear advantage of our relativistic calculation is that the resulting magnetic moments are quite insensitive to the quark masses. For example, we see by comparing Table I with the second column in Table II that the proton magnetic moment changes from 2.42 μ_N to 2.76 μ_N (a 14% enhancement) when the inverse quark mass is raised by 50%.

It is useful to discuss our results in light of the work of Chung and Coester [8], who fit their relativistic calculation of the moments to a linear function of β/m_q, with m_q the light quark mass, with the result (in units of nuclear magnetons):

$$\mu_p - 1 \simeq \frac{M_N}{m_q}\left\{\frac{2}{3}\left(1 - 0.19\frac{\beta}{m_q}\right) + \left[\frac{4}{3}F_{2u}(0) - \frac{1}{3}F_{2d}(0)\right]\left(1 - 0.097\frac{\beta}{m_q}\right)\right\}$$

$$\mu_n \simeq \frac{M_N}{m_q}\left\{-\frac{2}{3}\left(1 - 0.225\frac{\beta}{m_q}\right) + \left[\frac{4}{3}F_{2d}(0) - \frac{1}{3}F_{2u}(0)\right]\left(1 - 0.097\frac{\beta}{m_q}\right)\right\}, \tag{20}$$

where F_{2q} is the anomalous magnetic moment of the quark. Note that these formulae agree with the relations in Eqs. (1) in the limit $\beta/m_q \to 0$ and when $M_N = 3m_q$ if we take $\mu_q = [e_q + F_{2q}(0)]e/(2m_q)$, where $e_u = +2/3$, $e_d = -1/3$, etc. In these formulae the terms proportional to β/m_q are corrections from relativity to the contributions of the Dirac and Pauli moments of the quarks.

Relativistic effects tend to reduce the baryon magnetic moments, the primary effect coming from Melosh rotations of the quarks. This fact has been known for some time [21], and is reflected in factors like $(1 - k\beta/m_q)$ in Eq. 20. Lowering the quark mass raises the

quark Dirac moment $e_q/2m_q$ but lowers this factor, with the result that the moment of the baryon is largely unaffected, as seen in our calculated results.

In Ref. [9] essentially the same procedure was adopted, but with more sophisticated configuration-mixed wavefunctions resulting from a global fit to the spectrum [18]. These wavefunctions tend to have larger relativistic effects and so the nucleon anomalous moments, as we can see from Eq. (20), are reduced in magnitude, with the reduction in the neutron moment being larger than that in the proton moment. This is offset by adopting an anomalous magnetic moment for the down quark of $F_{2d}(0) =: \kappa_d = -0.153$ and a smaller moment for the up quark of $F_{2u}(0) =: \kappa_u = 0.085$. Note that these are substantial anomalous moments, yielding quark moments of $\mu_u = 1.13(+\frac{2}{3})\frac{e}{2m_q}$ and $\mu_d = 1.46(-\frac{1}{3})\frac{e}{2m_q}$.

When we adopt baryon wavefunctions which all have the same spatial size the SU(6) symmetry represented by the relations from Eqs. (1) for the octet baryon magnetic moments is broken, but those relations partially apply to our relativistic decuplet baryon results. This is because the spin and spatial wavefunctions of the decuplet baryons have separate total permutational symmetry. One can think of each quark as providing an effective additive moment, as defined by Eqs. (1). The effective additive moment of a quark depends on its environment, which in this case means the masses of the other quarks in the baryon. Fitting Eqs. (1) to our relativistic results in the second column of Table II yields the effective additive quark moments $\mu_u = 1.82$, $\mu_d = -0.91$ in the Δ states, $\mu_u = 1.77$, $\mu_d = -0.89$, and $\mu_s = -0.63$ in the Σ^* states, $\mu_u = 1.75$, $\mu_d = -0.88$, and $\mu_s = -0.62$ in the Ξ^* states, and $\mu_s = -0.61$ in the Ω^-. Note that in all cases the effective additive moments of the light quarks are in the ratio of their charges. There is a slight dilution of the effective moments of the quarks when in the environment of heavier quarks.

Our results appear to be consistent with those of McKellar *et al.* [6], who choose not to adopt anomalous moments for the quarks, but instead concentrate on examining the dependence of the moments on the masses of the light and strange quarks and the oscillator parameter β. They conclude that there is no choice of β and light-quark mass for which the nucleon moments are reproduced, and that a fit to the octet data is not possible without

the inclusion of quark anomalous moments.

In the third column of Table II we have shown the result of fitting the nine precisely determined magnetic moments by allowing nonzero anomalous magnetic moments of the quarks (we did not allow the masses to vary from the values prescribed by Refs. [18–20], or vary the harmonic oscillator constant from the value 2.08 fm^{-1}). This three-parameter fit reduces the root-mean-square deviation of the calculated moments from the nine precise data from 0.135 μ_N to 0.076 μ_N. The resulting quark anomalous moments are rather small: $\kappa_u = -0.011$, $\kappa_d = -0.048$, and $\kappa_s = -0.020$ in units of quark magnetons [for each flavor of quark $\kappa_q = F_{2q}(0)$, so that κ_q is in units of $\mu_q = e/2m_q$, where e is the electron charge, see Eq. (4)]. Equivalently, if we define a quark moment by $\mu_q = [F_{1q}(0) + F_{2q}(0)]\frac{e}{2m_q}$, we have $\mu_u = 0.98(+\frac{2}{3})\frac{e}{m_u}$, $\mu_d = 1.14(-\frac{1}{3})\frac{e}{m_d}$, and $\mu_s = 1.06(-\frac{1}{3})\frac{e}{m_s}$.

In the fourth column of Table II, we have shown the effects of adopting a simple nonrelativistic dependence of the harmonic oscillator size parameters on the masses of the quarks in the baryon, *i.e.*

$$\beta_\rho = (3Km)^{\frac{1}{4}}, \quad \beta_\lambda = (3Km_\lambda)^{\frac{1}{4}}, \tag{21}$$

where $m_\rho = m_1 = m_2$ is the mass of the two equal mass quarks and

$$m_\lambda = \frac{3mm_3}{2m + m_3}, \tag{22}$$

where m_3 is the mass of the odd quark out. For strangeness-zero baryons we have used $\beta_\rho = \beta_\lambda = 0.410$ GeV when $m = m_3 = 0.220$ GeV, so solving for $3K$ (K is the oscillator constant) we find $3K = 0.128\ \mathrm{GeV}^3$. Note that this differs from the approach taken by Schlumpf [7,11] who concentrated on calculating weak decay constants in terms of strongly asymmetric β parameters, which imply significant diquark clustering in the baryon wavefunctions. Such strong clustering in the wavefunctions does not arise in the relativized quark model of the spectrum of these states [18].

As can be seen from Table II, our results are largely insensitive to changes in the wavefunction, here made through the simple mechanism of changing the harmonic oscillator scale.

This can be easily understood, as this scale only enters in the relativistic corrections to the formulae (1). Equivalently, details of the wavefunction can only affect the size of the change in the moments due to relativistic corrections illustrated in Table I. However, a slightly better fit to the moments is obtained when using these wavefunctions, with the result that the r.m.s. deviation from the data for the nine precisely measured moments is reduced to 0.068 μ_N. The quark anomalous moments that give this fit are essentially unchanged at $\kappa_u = -0.006$, $\kappa_d = -0.047$ and $\kappa_s = -0.022$.

Aznauryan *et al.* [5] performed a fit to the 1984 data for the moments of the octet baryons using anomalous moments for the quarks, and make predictions for weak decay constants. They adopt quark masses of $m_{u,d} = 271$ MeV and $m_s = 397$ MeV, and allow the parameter β in the Gaussian wavefunction $\exp[-M_0^2/\beta^2]$ to vary linearly with the average mass of the three quarks. Note that this choice of wavefunction does not allow β_ρ and β_λ to differ, as is required by the solution of the dynamical problem. Their fit to the octet moments is of similar quality to ours, with the exception of the $\Sigma^0 \to \Lambda^0$ transition moment, which is too small. The quark anomalous moments they find for their fit are $\kappa_u = 0.012$, $\kappa_d = -0.059$, and $\kappa_s = -0.016$, similar to those found here.

Our results for the decuplet baryon moments differ from those of Schlumpf [11] due to our adoption of different quark masses. Schlumpf uses $m_u = m_d = 260$ MeV in his relativistic fits, which is significantly *larger* than our 220 MeV, and an m_s of 380 MeV which is *smaller* than our value of 419 MeV. The same is true of the quark masses used by Aznauryan *et al.* Our quark masses are motivated by relativized fits to the meson and baryon spectra [18,19], and to isospin violations in ground state meson and baryon masses. A difference between the light and strange quark masses of 120 MeV may not be large enough to be consistent with these other constraints.

The use of quark anomalous magnetic moments is intended to account for the fact that constituent degrees of freedom are effective, and that such quarks receive substantial QCD dressing. From the point of view of chiral symmetry, one could express such quark dressing in terms of pion loops. Several groups have investigated this possibility [22–24]. In general,

the anomalous moments which result from pion loops are significantly higher than those obtained in our phenomenological fit. For example, recent results by Ito [24] give the ranges κ_u = 0.0550–0.1118 and κ_d = -(0.0832–0.1438). His κ_u has the opposite sign from our phenomenological fit, and his κ_d is several times our value. Using his values of κ_u and κ_d to compute baryon moments would give a worse fit than one with no anomalous moments. On the other hand, Cohen and Weber find large cancellations of pion loop contributions with other corrections to baryon magnetic moments [22]. The lesson from this is that a constituent quark model is not easily corrected by adding pion loops to describe QCD quark dressing.

The physics of meson cloud effects in baryons was also studied from a general perspective of chiral symmetry by Cheng and Li [25,26]. They characterize effects of the quark sea in terms of parameters g_8 and g_0, corresponding to the contributions of $SU(3)_f$ pseudoscalar octet and singlet Goldstone bosons, respectively. They find a reasonable fit [25] to the measure $\Delta\Sigma$ of the quark contribution to the proton spin appearing in the Bjorken [27] and Ellis-Jaffe [28] sum rules, as well as to the magnetic moments of the baryon octet [26].

Such effects of the quark sea should also be considered in a relativistic quark model. Technically, the procedure will be much more difficult since, as we have seen, the quark moments do not enter in a simple additive fashion. Ma and Zhang find substantial reductions of the proton spin matrix elements, related to the axial coupling g_A, which enter the Bjorken [27] and Ellis-Jaffe [28] sum rules [29,30]. This property has also been noted by other authors [31]. Thus, the combined effects of relativity and the quark sea must be considered together. In that regard, it may be better to use a variation of the approach of Cheng and Li to parameterize the effect of the quark sea, rather than to compute specific meson loop contributions.

IV. CONCLUSIONS

We have seen that it is possible to achieve a quite satisfactory fit to the measured ground state baryon magnetic moments with the inclusion of relativistic effects and allowing the constituent quarks to have nonzero anomalous moments. This seems reasonable given that the constituent quark is an effective degree of freedom, much like the nucleon when it is bound into a nucleus. A measure of the effectiveness of our model is to compare to a fit using the simple additive model of Eqs. (1) and three arbitrary quark moments. The result of doing this is shown in Table II, with a root-mean-square deviation for the nine data of 0.100 μ_N. Clearly our relativistic fits improve on this. The differences are caused by the fact that, in a relativistic model, the moments of the quarks do not simply add to the moments of the baryons, as has been pointed out by many other authors [5–9,11].

The anomalous moments obtained in our fit can be thought of as quark sea effects which dress the effective degrees of freedom in such models. However, quark sea effects are not interchangeable with pion loop effects, as the actual numbers obtained in our fit differ considerably from a direct calculation of baryon moment modifications due to pion loops.

Our results are comparable to the fit to the moments achieved by Karl [1] in the presence of a strange quark contribution to the magnetic moment of the nucleons. The root-mean-square deviation for his fit to the eight octet moments is 0.084 μ_N. We would conclude that it is possible to improve on a simple nonrelativistic fit to the data without strange quark contributions to the magnetic moment of the nucleons.

The results presented here can be expected to change with the adoption of mixed wavefunctions such as those of Ref. [18] used by Cardarelli *et al.*, [9]. We have made some exploratory calculations of this type for the baryons for which data for their moments exist, and have found that it is impossible to achieve a fit of the quality shown in Table II with the relativized model wavefunctions of Ref. [18]. On the other hand, the results of Ref. [9] also reveal that that these wavefunctions have large amplitudes at higher momentum, which may lead to an overprediction of relativistic effects. In Ref. [9], this behavior was offset by

giving the quarks quite soft momentum-dependent form factors. Another possibility is to consider quark wavefunctions which have smaller amplitudes at higher momentum. This question is presently under investigation.

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VI. APPENDIX A: THE Σ^0 TO Λ^0 TRANSITION MOMENT

In Ref. [3] the transition moment for Σ^0 to Λ^0 is defined to be

$$\left[\frac{\mu_{\Sigma\Lambda}}{\mu_N}\right]^2 = \frac{1}{\tau}\frac{8\hbar M_p^2 M_\Sigma^3}{\alpha(M_\Sigma^2 - M_\Lambda^2)^3}, \tag{23}$$

where τ is the lifetime of the Σ^0 (which goes almost 100% through Λ^0 γ). In the Particle Data Group [16] the photon width of a resonance R decaying to a nucleon is given by

$$\Gamma_\gamma = \frac{k^2}{\pi}\frac{2M_N}{(2J+1)M_R}\left[|A_{\frac{1}{2}}|^2 + |A_{\frac{3}{2}}|^2\right], \tag{24}$$

where J is the spin of the decaying resonance and k is the photon c.m. decay momentum. Adapting this to the Σ^0 to Λ^0 decay, we have

$$\Gamma = \frac{k^2}{\pi}\frac{M_\Lambda}{M_\Sigma}|A_{\frac{1}{2}}|^2, \tag{25}$$

Replacing the lifetime in the first equation by $\frac{\hbar}{\Gamma}$, we have

$$\left[\frac{\mu_{\Sigma\Lambda}}{\mu_N}\right]^2 = \frac{M_p^2\Gamma}{\alpha k^3}, \tag{26}$$

where $k = (M_\Sigma^2 - M_\Lambda^2)/2M_\Sigma$ is the c.m. frame decay momentum.

The result is that we can write the transition moment directly in terms of the helicity amplitude (calculated in the rest frame of the decaying Σ^0) as

$$\left[\frac{\mu_{\Sigma\Lambda}}{\mu_N}\right]^2 = \frac{2M_p^2 M_\Lambda}{\pi\alpha(M_\Sigma^2 - M_\Lambda^2)}|A_{\frac{1}{2}}|^2. \tag{27}$$

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TABLES

TABLE I. Relativistic effects in baryon magnetic moments for which data exist; here $m_{u,d} = 330$ MeV, $m_s = 550$ MeV, and $\beta_\rho = \beta_\lambda = 2.08$ fm^{-1}. Moments are in units of nuclear magnetons $\mu_N = e/2m_N$. Data are from Ref. [16].

moment	nonrel.	rel.	data
μ_p	2.85	2.42	2.79
μ_n	-1.89	-1.34	-1.91
μ_Λ	-0.61	-0.50	-0.61
μ_{Σ^+}	2.72	2.22	2.46
μ_{Σ^-}	-1.07	-1.00	-1.16
$\mu_{\Sigma^0\to\Lambda^0}$	-1.62	-1.20	-1.61
μ_{Ξ^0}	-1.39	-1.09	-1.25
μ_{Ξ^-}	-0.44	-0.63	-0.65
$\mu_{\Delta^{++}}$	5.69	5.05	3.5-7.5
μ_Ω	-1.71	-1.77	-2.02

TABLE II. Baryon magnetic moments calculated in the relativistic model with $m_{u,d} = 220$ MeV and $m_s = 419$ MeV. In the second and third columns β_ρ and β_λ are fixed at 2.08 fm^{-1}, while in the fourth column they vary according to the simple formula of Eq. (21). The third and fourth columns are independent fits to the moments using the three quark anomalous moments, as described in the text. A nonrelativistic fit using three quark moments and Eqs. (1) is shown in the first column for comparison purposes. Moments are in units of nuclear magnetons $\mu_N = e/2m_N$.

moment	NRQM fit	$\kappa_i = 0$ $\beta_\rho = \beta_\lambda$	κ_i fit $\beta_\rho = \beta_\lambda$	κ_i fit β_ρ, β_λ from Eq. (21)	data
μ_p	2.66	2.76	2.76	2.78	2.79
μ_n	-1.94	-1.62	-1.82	-1.82	-1.91
μ_Λ	-0.69	-0.61	-0.65	-0.63	-0.61
μ_{Σ^+}	2.54	2.56	2.52	2.51	2.46
μ_{Σ^-}	-1.14	-1.08	-1.29	-1.29	-1.16
$\mu_{\Sigma^0\to\Lambda^0}$	-1.58	-1.45	-1.54	-1.52	-1.61
μ_{Ξ^0}	-1.45	-1.31	-1.35	-1.32	-1.25
μ_{Ξ^-}	-0.53	-0.63	-0.63	-0.64	-0.65
$\mu_{\Delta^{++}}$		5.45	5.33	5.38	3.5-7.5
μ_{Δ^+}		2.72	2.47	2.51	
μ_{Δ^0}		0.01	-0.36	-0.34	
μ_{Δ^-}		-2.72	-3.22	-3.21	
$\mu_{\Sigma^{*+}}$		2.91	2.79	2.84	
$\mu_{\Sigma^{*0}}$		0.25	0.01	0.03	
$\mu_{\Sigma^{*-}}$		-2.41	-2.77	-2.78	
$\mu_{\Xi^{*0}}$		0.51	0.40	0.42	
$\mu_{\Xi^{*-}}$		-2.12	-2.36	-2.38	
μ_Ω	-1.95	-1.84	-1.96	-1.99	-2.02
$\sqrt{\sum(\mu_{\rm th}-\mu)^2/9}$	0.100	0.135	0.076	0.068	